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TITLE- Digital Interpolation and Magnification of Pictures

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DATE- June 25, 1968

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AUTHOR(S)- H. A. Helm

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## ABSTRACT

The process of magnification of digitized pictures is analyzed and three different methods are described. All depend upon interpolation. One does the interpolation in both dimensions on a digital computer. Another uses the computer for interpolation in the vertical direction and analog filters in the horizontal direction. Finally, and most interesting, an all analog system is proposed using a laser optical system.

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SUBJECT: Digital Interpolation and Magnification of Pictures - Case 101

DATE: June 25, 1968

FROM: H. A. Helm

TM-68-1033-4

# TECHNICAL MEMORANDUM

## INTRODUCTION

The pictures from the Lunar Orbiter exist as analog video recordings and as two-dimensional arrays of numbers each of which represents the brightness of the original scene at that point or sample. This memorandum will be concerned only with the latter form of video data. These points are sufficiently close together so that when they are reconverted to brightness in a digital to video converter a visually satisfactory reconstruction of the scene is obtained. If, however, it is desired to magnify a subarea of the picture there are two choices. First, one can take the smaller set of points covering the desired area and display these over the original area. This results in an extreme granularity for larger magnifications which is unsatisfactory. Second, one can photograph the converter tube and enlarge the resulting negative. Characteristically, this is done with 35 mm film but this second photography introduces noise. A typical 35 mm negative of an original scene will, with nominal care in processing, give a good 8 x 10 print. Any print larger will show definite grain. Thus, the original scene is processed quite satisfactorily but optical enlargement introduces appreciable additional noise into the system. For this reason, the first method is considered in this memorandum with the additional feature that interpolation is done between the points of scene to be magnified. This interpolation is done in such a way that no information present in the original sampled input is lost. Three methods can be used to implement the interpolation: all digital; digital in the vertical direction and analog across a scan line; all analog using laser techniques. These methods are covered in detail below.

## REVIEW OF THEORY

The interpolation algorithm is an extension to two-dimensions of Shannon's sampling theorem. The basic application of Fourier analysis to photographic processes is reviewed in TM-67-1033-3, A Digital Filter Method for the Improvement of Photographic Resolution, by L. D. Nelson. The extension to two dimensions of Shannon's sampling theorem has been done by Cheng [1]. Some of his results are repeated below for convenience and continuity.

The Fourier transform of a picture function p(x,y) representing the brightness of some scenes as a function of the orthogonal coordinates x,y is:

$$P(u,v) = \iint_R p(x,y)e^{i2\pi(ux+vy)}dx\ dx \tag{1}$$

where u,v are the spatial frequencies of the scene commonly given in cycles/millimeter and R is the region over which the scene is defined. In the rigorous theory R is infinite, but in any real scene sharply limited. Practically, this difference leads to "edge effects" which are generally ignored. The inverse Fourier Transform is:

$$p(x,y) = \int_{-U}^{U} \int_{-V}^{V} P(u,v)e^{-i2\pi(xu+yv)}du\ dv \tag{2}$$

The finite spatial frequency limits in (2) represent the same approximation to the physical system that an ideal bandpass filter characteristic does to a physically realizable network.

Equations (1) and (2) define a spectral theory for optical systems. Following Cheng, we desire to express p(x,y) in a Fourier expansion:

$$p(x,y) = \sum_{ij} C_{ij}\ \Phi_{ij}\ (x,y) \tag{3}$$

where the $\Phi_{ij}$ are some orthonormal functions and:

$$C_{ij} = \iint_R p(x,y)\ \Phi_{ij}\ (x,y)\ dx\ dy \tag{4}$$

We desire to pick the $\Phi$'s so that the $C_{ij}$'s are as easy to compute as possible. In fact, we want $C_{ij}$ to be value of the scene at some particular point $(x_i, y_j)$. Thus,

$$p(x_i, y_j) = \iint_R p(x,y)\, \Phi_{ij}(x,y)\, dx\, dy \tag{5}$$

The solution to the integral equation (5) is straightforward and is given by

$$\Phi_{ij} = \frac{B \sin\left[\frac{\pi}{\Delta x}(x-x_i)\right] \sin\left[\frac{\pi}{\Delta y}(y-y_j)\right]}{(x-x_i)\ (y-y_j)} \tag{6}$$

where $\Delta x = \frac{1}{2U}$ ; $\Delta y = \frac{1}{2V}$ ; and B is a normalization constant. Obviously, $\Delta x$ corresponds to the Nyquist interval in the x direction, $\Delta y$ in the y direction. If the points $(x_i, y_j)$ are equally spaced in integral multiples of $\Delta x$ in the x direction and $\Delta y$ in the y direction then the $\{\Phi_{ij}\}$ are an orthonormal basis for the picture space.

Ideally, in space applications the sampling points $x_m = x_0 + m\Delta x$; $m=0, 1, 2, \ldots, M$, $y_n = y_0 + n\,\Delta\, y$; $n = 0, 1, 2, \ldots, N$ would be chosen from a knowledge of the spectral characteristics of the pictures to be transmitted. Unfortunately, in practice the two-dimensional bandwidth of the original scene is rarely known and other considerations usually dictate the choice of the sampling interval. However, once the information exists in sampled form one can only recover those frequencies less than that frequency U (or V) for which $\Delta x$ (or $\Delta y$) is the Nyquist interval but these can be recovered completely by means of (6). Thus, for each point in a given subset of some scene which is to be enlarged one can interpolate as many points as one desires so that the reconstruction is visually satisfactory. That is the "granularity" can be made arbitrarily small and enlargement can be made to the entire limit of the information contained in the sampled picture.

## ALL DIGITAL AND DIGITAL-ANALOG INTERPOLATION

Since the brightness of each sample point is encoded as a digital number, the obvious way to interpolate is on a computer. A sufficiently accurate approximation to the transcendental equation (6) is used to compute any desired number of interpolated values as shown in Figure I. This is not exactly economical of computer time since (6) involves two variable approximations. However, some digital to video converters (such as that at BTL, Murray Hill) have provision for an analog interpolation along the horizontal direction. This is accomplished by sweeping the beam of the CRT horizontally a fixed length of time and bringing the video samples in at fixed intervals of time determined by the number of samples per horizontal scan. These samples are read as binary numbers from a computer tape, converted in a DAC to voltage and applied through delay lines and networks to intensity modulate the CRT. The delays and filters have a voltage output which closely approximates a $\frac{\sin[\omega(x-x_i)]}{[x-x_i]}$ function. Thus, in the horizontal direction the picture shows no granularity due to sampling. Since, (6) is the product of two such functions the digital computer needs only to interpolate in the vertical direction leading to the computation pattern shown in Figure II. This pattern saves (K) (K+1) (M) (N) + MK interpolations where K is the number of interpolations within each sample interval assumed equation the x, and y directions.

ALL ANALOG INTERPOLATION

Consider the laser optical system shown in Figure III. A transparency of some scene p(x,y) is illuminated by the coherent light of the laser at one focal point of lens 1. At the other focal point the Fourier Transform, P(u,v) will appear [3] which is also at a focal point of lens 2. The image of p(x,y) will again appear at the second focal point of lens 2. Now supposing we use a computer or some other device to derive the Fourier Transform G(u,v) of a compensating filter g(x,y). If we make a transparency of it and place it in the Fourier plane of the system then the image formed, b(x,y), will be the inverse transform of the product, P(u,v) G(u,v). It is well known, [2], that this is also the convolution:

$$b(x,y) = \iint_R p(x,y)\, g(x-\xi,\, y-\mu)\, d\xi\, d\mu \tag{7}$$

of p(x,y) and g(x,y). What is desired is the transform whose inverse will give us a $\frac{\sin x}{x}$ type of function in the image plane.

Consider now, an array of discrete sample points in the scenic plane and assume they are spaced at the Nyquist intervals, $\Delta x = \frac{1}{2U}$, $\Delta y = \frac{1}{2V}$. Let each point be represented by the amplitude of the delta function; $p_{ij}\ \delta(x-x_i,\, y-y_j)$ (more properly the generalized function or distribution). For convenience let $p_{ij} = 1$. Now the two dimensional F.T. of $f(x-x_i, y-y_j)$ is given [2] by:

$$f(x-x_i,\ y-y_j) \xrightarrow{\text{F.T.}} F(u,v)\ e^{i2\pi(ux_i+vy_j)} \qquad (8)$$

Where the region R of (1) is infinite.

Thus,

$$\delta(x-x_i,\ y-y_j) \xrightarrow{\text{F.T.}} e^{i2\pi(ux_i+vy_j)}$$

In the Fourier plane a rectangular aperture is placed as shown in Figure IV. Consider first the single point source $\delta(x-x_i,\ y-y_j)$. Since the aperture in the Fourier plane limits the frequency response, the inverse F.T. $b(x,y)$ is:

$$\int_{-U}^{U}\int_{-V}^{V} e^{-i2\pi(xu + yv)}\ e^{i2\pi(ux_i + vy_j)}\ du\ dv$$

$$= \int_{-U}^{U} e^{i2\pi ux_i}\ \frac{\sin[V2\pi(y-y_j)]}{\pi(y-y_j)}\ du$$

$$= \frac{1}{\pi^2}\ \frac{\sin[\frac{\pi}{\Delta x}(x-x_i)]\ \sin[\frac{\pi}{\Delta y}(y-y_j)]}{(x-x_i)\quad(y-y_j)} \qquad (9)$$

which is as desired for a single point source at $(x_i, y_j)$. For the entire array of point sources each of amplitude $p_{ij}$, equation (7) becomes a finite sum and the output becomes:

$$b(x,y) = B \sum_{ij} p_{ij}\ \frac{\sin[\frac{\pi}{\Delta x}(x-x_i)]\sin[\frac{\pi}{\Delta y}(y-y_j)]}{(x-x_i)\quad(y-y_i)} \qquad (10)$$

as desired.

Suppose now that to obtain magnification we increase the spacing between samples to $\Delta x' = a\Delta x$ and $\Delta y' = b\Delta y$ then to maintain our interpolation we must change the aperture in the Fourier plane to:

$$U' = \frac{1}{2\Delta x'} = \frac{1}{2a\Delta x} = \frac{U}{a}, \quad V' = \frac{V}{b}$$

This may also be shown more elegantly by the known property of F.T.'s for magnification:

$$f(ax,by) \xrightarrow{\text{F.T.}} \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

with a simple change of variable.

CONCLUSION

Three different schemes have been described for the enlargement of digitized pictures. The method of using a computer for interpolation in the vertical direction and analog filters in the horizontal is certainly feasible and the equipment for this is available. However, the third method is extremely interesting and offers the possibility of very rapid batch processing of data. This could be extremely valuable to the space program.

1033-HAH-lf

H. A. Helm

Attachments
References
Figures I - IV

## REFERENCES


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2. Arsac, J., Fourier Transforms and the Theory of Distributions, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1966.

3. Cutrona, L. J., Leith, E. N., Palermo, C. J., and Porcello, L. J., Optical Data Processing and Filtering Systems, IRE Transactions on Information Theory, Vol. IT-6, No. 3, June 1960, pp. 386-400.

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X Sampled video

o Digitally computed interpolation point

Figure I

ALL DIGITAL INTERPOLATION

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X • • • X • • • X • • • X • • • X
```

X Sampled video

o Digitally computed interpolation point

• Analog filter derived interpolation point

Figure II

ANALOG DIGITAL INTERPOLATION

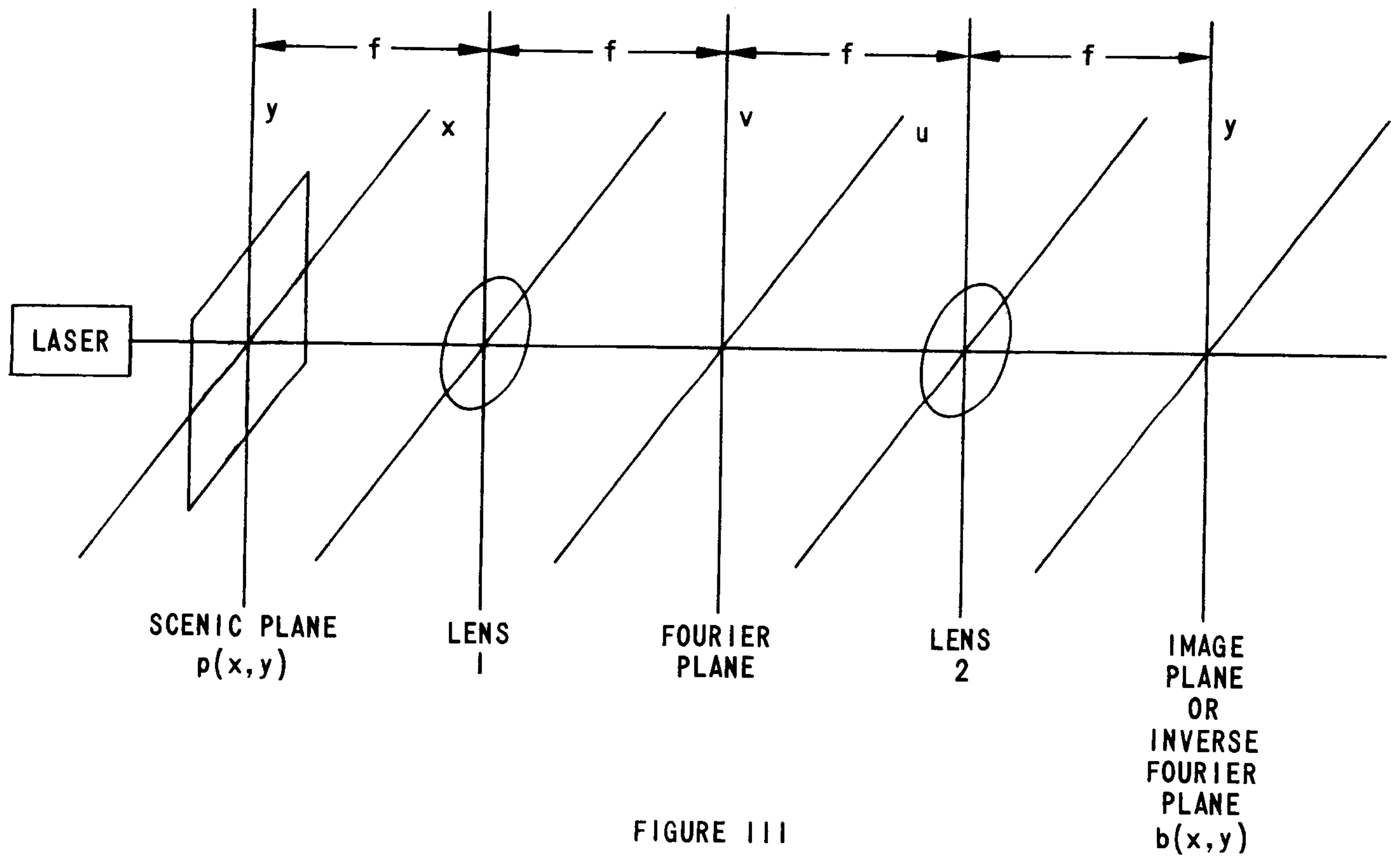
f
f
f
f
y
x
v
u
y
x
LASER
SCENIC PLANE
p(x,y)
LENS
1
FOURIER
PLANE
LENS
2
IMAGE
PLANE
OR
INVERSE
FOURIER
PLANE
b(x,y)

FIGURE III

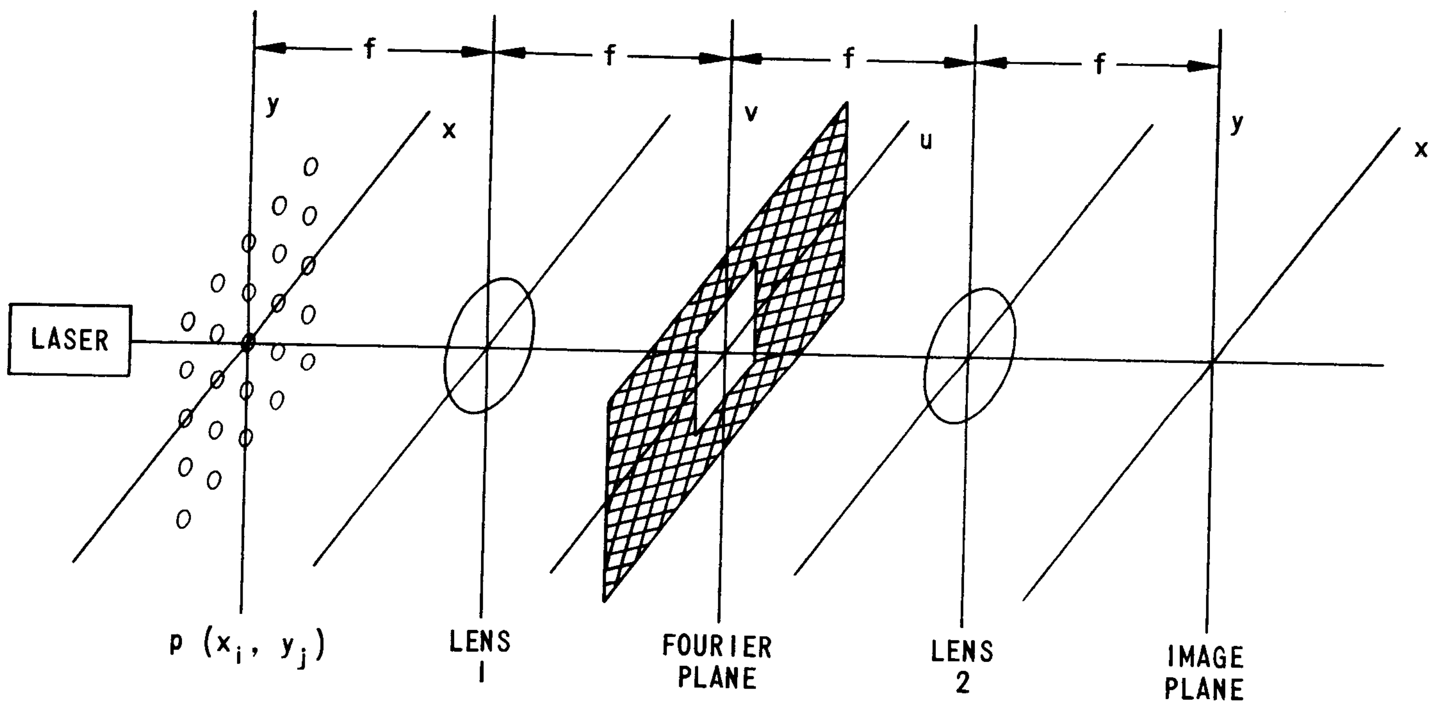
f
f
f
f
y
x
v
u
y
x
LASER
p $(x_i, y_j)$
LENS 1
FOURIER PLANE
LENS 2
IMAGE PLANE

FIGURE IV